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II. Solution by the PROPOSER.

Let, $x^m+\frac{m(m-1)}{2}x^{m-2}y^2+\ldots.+\frac{m(m-1)}{2}x^2y^{m-2}+y^m$, and $mx^{m-1}y+\frac{m(m-1)(m-2)}{2.3}x^{m-3}y^3+\ldots.+\frac{m(m-1)(m-2)}{2.3}x^3y^{m-3}+mxy^{m-1}$ be the numbers.

Then $(x+y)^m$, $(x-y)^m$, $(x^2-y^2)^m$, is their sum, their difference, and the difference of their squares. m must be divisible by 2, 3, and 4; this is the case when $m=12$. Then the numbers are $x^{12}+66x^{10}y^2+\ldots+66x^2y^{10}+y^{12}$,and $12x^{11}y+220x^9y^3+\ldots.+220x^3y^9+12xy^{11}$ and $(x+y)^{12}$, $(x-y)^{12}$, $(x^2-y^2)^{12}$ is their sum, their difference, and the difference of their squares.

Let $x=2$, $y=1$, then the numbers are 265721, 265720. Their sum $=(3)^{12}=531441=(729)^2=(81)^3=(27)^4$. Their difference$=1=(1)^2=(1)^3=(1)^4$. Difference of their squares$=531441=(729)^2=(81)^3=(27)^4$.

Many other numbers can be found satisfying the conditions.

Also solved by *H. W. Draughon, and J. F. W. Scheffer.*

21. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Find (1) nine positive *integral numbers* in arithmetical progression the sum of whose squares is a *square number*; and (2) find nine integral *square numbers* whose sum is a *square number*.

I. Solution by H. W. DRAUGHON, Olio, Mississippi.

(1). Let, $x+4y$, $x+3y$,$x+2y$, $x+y$, x,$x-y$,$x-2y$,$x-3y$, and $x-4y$ be the numbers. Then we are to make the sum of their squares, $9x^2+60y^2=\square$. Let us assume, $9x^2+60y^2=(3x+6m)^2=9x^2+36mx+36m^2$; then,

$x=\frac{60y^2-36m^2}{36m}=\frac{5y^2}{3m}-m$. In order that x may be integral put, $y=3pm$...(1).

Then, $x=15p^2m-m=(15p^2-1)m$....(2). p and m can have any positive, integral values that will make $x>4y$.

Let us make, $p=1$ and $m=1$; then, $x=14$, $y=3$, and the numbers are, 2, 5, , 11, 14, 17, 20, 23, and 26. The sum of their squares is $(48)^2$.

Again, let, $p=2$, and $m=1$; then, $x=59$, $y=12$, and the numbers are, 11, 23, 35, 47, 59, 71, 83, 95, and 107. The sum of the squares of this set is $(183)^2$. An infinite number of sets can be thus obtained from (2).

(2). In the *Mathematical Messenger*, Vol. 7, No. 5. page 47, I find the following formula for n square numbers whose sum is a square:

$S+\frac{1}{4}(p-q)^2=\frac{1}{4}(p+q)^2$, in which $S=pq$=the sum of $n-1$ square numbers. Here, $n=9$. Let us assume, $S=(3)^2+(4)^2+(5)^2+(8)^2+(9)^2+(10)^2+(11)^2+(12)^2=560=10\times56$. Let us make $p=56$ and $q=10$; then, we have, $S+(23)^2=(33)^2$. Any other factors of 560 may be taken. When factors give fractional results we clear of fractions.

II. Solution by R. J. ADCOCK, Larchland, Illinois.

$a, a+x, a+2x, a+3x, \ldots . a+(n-1)x$, is an arithmetical progression of n terms, their sum of squares is $a^2+(a+x)^2+(a+2x)^2+(a+3x)^2+\&c\ldots.$ $+[a+(n-1)x]^2=S=na^2+(n^2-n)ax+\frac{1}{6}(2n^3-3n^2+n)x^2$ by method of difference. This sum is made=a rational square by the method given in Encyclopedia Britannica, Vol. I, Algebra article 121, 9th edition. When n is a square number $na^2+(n^2-n)ax+\frac{1}{6}(2n^3-3n^2+n)x^2=(n^{\frac{1}{2}}a+hx)^2$ $=na^2+2ahn^{\frac{1}{2}}x+\frac{1}{6}(2n^3-3n^2+n)x^2$. Then $x=\dfrac{2ahn^{\frac{1}{2}}-(n^2-n)a}{\frac{1}{6}(2n^3-3n^2+n)-h^2}=\frac{3}{2}$ for $a=1$, $n=9$, $h=14$. The algebraic value of x in expressions for S_1 after clearing denominatives gives a general solution, the numerical $\frac{3}{2}$, gives $2^2+5^2+8^2+11^2+14^2+17^2+20^2+23^2+26^2=48^2$.

Also solved by *F. P. Matz. G. B. M. Zerr, J. H. Drummond, C. D. Schmitt, J. Scheffer, and M. A. Gruber*

PROBLEMS.

30. Proposed by COOPER D. SCHMITT, Knoxville, Tennessee.

A and B are two integers, A consisting of $2m$ figures each being 1, and B consisting of m figures each being 4. Prove that $A+B+1$ is a square.

31. Proposed by M. A. GRUBER, War Department, Washington, D. C.

How many scalene triangles, of integral sides, can be formed with an altitude of 12? How many isosceles triangles?

AVERAGE AND PROBABILITY.

Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

17. Proposed by A. L. FOOTE, No. 80, Broad St. New York.

A person 30 years of age has an annuity for 10 years, the present worth of which is $1000, provided he lives but ten years; for, if he dies, the annuity ceases What is the annuity worth, on the assumption that 75 out of every 4385 persons die annually, between the ages 30 and 40 years?

Solution by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Kidder Institute, Kidder, Missouri.

Let S=the annuity, the present value, P, of which, for 10,$=n$, years is $100.